



Valuation of insurers' contingent capital with counterparty risk and price endogeneity



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ABSTRACT

This study develops a structural framework to value insurers' contingent capital with counterparty risk (CR) and overcomes the problem of price endogeneity (PE) in the valuation model. Our results on the focal contingent capital instrument – catastrophe equity put option (CatEPut) – indicate that prices can be significantly overestimated without considering CR and be significantly underestimated without considering PE. This study also examines how CatEPuts affect the buyer's probability of default (PD). Our results show that buying a CatEPut lowers the PD for high-risk insurers, but not necessarily so for low-risk insurers; however, without taking CR and PE into account, one may significantly overestimate the credit enhancement provided by the CatEPuts.

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1. Introduction

The number of yearly catastrophe events, including both natural catastrophes and man-made disasters, has grown steadily from less than 100 in 1970 to 304 in 2010, with catastrophes claiming nearly 304,000 victims and costing insurers more than US\$43 billion in 2010 alone.² During the last four decades, insurance companies have been seeking solutions other than traditional reinsurance contracts to increase the capacity for catastrophe coverage and to diversify their catastrophe risk. The capital markets have developed alternative risk transfer instruments to provide (re)insurance companies with vehicles for hedging such catastrophe risk. These instruments can be broadly classified into three categories from the point of view of an insurer's balance sheet: asset instruments consist of catastrophe-linked derivatives such as catastrophe futures

and catastrophe options³; liability instruments include the most prominent type of catastrophe-linked security – catastrophe bonds⁴; and catastrophe-linked contingent capital contracts are equity instruments.

The catastrophe bond (CAT bond) is the most successful instrument to date, but its growth seems stagnant and is not as expected. The literature has proposed some explanations for its less-than-expected growth such as moral hazard (Lee and Yu (2002)), basis risk (Cummins et al. (2004)), asymmetric information (Finken and Laux (2009)), downside risk aversion among investors (Barrieu and Loubergé (2009)), and its fully collateralized feature (Lakdawalla and Zanjani (2012)). Barrieu and Loubergé (2009), however, suggest that a hybrid of contingent capital and CAT bonds may help to raise the volume of CAT bond issues.

Contingent capital has been used by insurance companies since the 1990s to raise capital in order to hedge against catastrophe risk. Basel III has recently proposed the design of contingent capital for banks to hedge against systemic risk and to reduce any exter-

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² See Swiss Re 2011 Sigma Report. The average worldwide losses from insured natural catastrophes were US\$5.1 billion per year in 1970–1989, but rocketed to US\$27.1 billion per year in 1990–2009.

³ See also Bakshi and Madan (2002), Cummins et al. (2004), and Chang et al., (2010).

⁴ See also Loubergé et al., 1999, Lee and Yu (2002), and Finken and Laux (2009).

nalities created by systemically important institutions.⁵ An insurer's contingent capital can be provided through contingent surplus notes (CSNs)⁶ or catastrophe equity puts (CatEPuts)⁷ by issuing debt or company shares at predetermined terms following a loss that exceeds a certain threshold. CSNs give the insurer the right to immediately obtain funds by issuing surplus notes at a predetermined interest rate. CatEPuts provide the buyer the option to issue a certain amount of new shares at a prenegotiated price. Both instruments supply the buyer with additional equity capital when funds are needed the most to cover its catastrophe losses.⁸ The literature on contingent capital for insurance companies has focused on CatEPuts, because the contract design of CSNs is relatively simple and can be considered as a special case of CatEPuts.

Cox et al. (2004) are the first to investigate the pricing model for CatEPuts. Their model assumes that only the catastrophe event affects the stock price, while the size of the catastrophe is irrelevant. Jaimungal and Wang (2006) extend the results of Cox et al. (2004) to analyze the pricing of CatEPuts under stochastic interest rates with catastrophe losses generated by a compound Poisson process. Lin et al. (2009) and Chang et al. (2011) focus on the stochastic nature of catastrophic intensity. Lin et al. (2009) assume catastrophe losses are generated by a doubly stochastic Poisson process with lognormal intensity, while Chang et al. (2011) assume a Markov Modulated Poisson process whose intensity depends on climate states.

This literature, however, fails to recognize counterparty risk and price endogeneity in the valuation of CatEPuts. CatEPuts differ from CSNs and CAT bonds in counterparty performance risk. Meyers and Kollar (1999) and Cummins (2008) point out that CatEPut transactions do not form a single purpose reinsurer (SPR) and are not collateralized. Therefore, CatEPuts expose the buyer to counterparty risk.⁹ Particularly, when the CatEPut buyer (e.g. an insurance company) faces large catastrophic losses and has the right to exercise the option, the CatEPut seller (e.g. a reinsurance company) may also assume large reinsurance claim payments and may not be able to fulfill its obligation to buy the shares at the specified price. In order to incorporate counterparty risk, this study follows Merton (1974) and sets up a structural model to value the CatEPuts.

We further point out and overcome an endogenous problem in the literature – that is, the price paid for acquiring the CatEPut by the buyer will in turn affect the payoff of the CatEPut and the price itself. This price endogeneity also exists in other applications of Merton's model such as in the deposit insurance literature, for example, Merton (1977) and Ronn and Verma (1986), and in the insurance guaranty literature, for example, Cummins (1988) and Duan and Yu (2005). When valuing the contingent-claim contract, either for deposit insurance or insurance guaranty, these authors all ignore the initial acquisition cost of the contract. However, Ahn et al. (1999) demonstrate that hedging costs play a critical role in risk management, and an optimal risk management problem must consider the trade-off between hedging costs and risk exposures. Moreover, a real world CatEPut price charged for LaSalle Re in 1997 was 2.35% (rate on line), which is not insignificant and thus the effect of price endogeneity can be considerable. When the con-

tingent-claim contract is not free, the purchase of the contract will change the asset value and impact the price. Hence, we propose a simple method to overcome this endogeneity and measure its influence on a fairly-priced premium.

In addition to the valuation framework, this study examines how a CatEPut transaction affects the default probability of its buyer.¹⁰ Since CatEPuts allow the buyer to sell new shares at a predetermined price when the put is triggered, the change in probability of default (PD) includes a pure put option effect and a new equity effect. The new equity effect helps to reduce the buyer's PD due to new capital infusion. However, the pure put option effect on PD is not so clear. We look further into the components of the pure put option effect and draw a conclusion on the change of the buyer's total default probability.

The rest of this study is organized into five sections. Section 2 shows the model assumptions and Section 3 develops a model to value CatEPuts. Section 4 presents the numerical analysis, justifies the parameters, and demonstrates the main results. Section 5 provides further discussions on the change in probability of default, correlation of catastrophe risk, and interest rate risk. Section 6 concludes the paper.

2. Assumption

Like most CatEPut transactions, we assume the buyer of CatEPuts is an insurance company (denoted by I) and the writer is a reinsurance company (denoted by R).

2.1. Interest rate

We assume that the insurer and reinsurer operate in an environment where interest rates are stochastic and follow the squared-root process of Cox et al. (1985), the CIR model hereafter, which avoids the negative interest rate that may appear in Vasicek's model.¹¹ The instantaneous interest rate process under the risk-neutral pricing measure \mathbb{Q} can be written as:

$$dr_t = \kappa(\mu_r - r_t)dt + \sigma_r\sqrt{r_t}dZ_{r,t}, \quad (1)$$

where r_t denotes the instantaneous interest rate at time t ; κ is the mean-reverting force measurement; μ_r is the long-run mean of the interest rate; σ_r is the volatility parameter for the interest rate; and $Z_{r,t}$ is a Wiener process under \mathbb{Q} .

2.2. Asset value

Following Merton (1977) and Cummins (1988), we assume that the asset values of the insurer ($A_{I,t}$) and reinsurer ($A_{R,t}$) are governed by geometric Brownian motions. The process under the risk-neutral measure \mathbb{Q} can be written as (at time t):

$$\frac{dA_{x,t}}{A_{x,t}} = r_t dt + \sigma_{A,x} dW_{x,t}, \quad x = I \text{ or } R, \quad (2)$$

where subscript $x = I$ for insurer, and $x = R$ for reinsurer; $\sigma_{A,x}$ is the total volatility of x -company's asset returns, and $W_{x,t}$ is a Wiener process under \mathbb{Q} . The correlation coefficient between $W_{x,t}$ and $Z_{r,t}$ is $\rho_{x,r}$.

2.3. Liability and catastrophe

We follow Cummins (1988), Duan and Yu (2005), and Gatzert and Schmeiser (2008) to assume that the total contractual liability

⁵ See also Pennacchi (2010), Hilscher and Raviv (2011), and Glasserman and Nouri (2012).

⁶ In the U.S., statutory accounting rules treat CSNs as equity on an insurer's statutory balance sheet, allowing an insurer to increase its capacity to write business.

⁷ A registered trademark of Aon Limited.

⁸ Nationwide issued the first CSN in 1995 and RLI issued the first CatEPut in 1996. Following these two, numerous transactions have been executed, such as by LaSalle Re (1997), Horace Mann (1997), Trenwick (2000), Nationwide (2004), Farmers (2007), and SCOR (2010).

⁹ The CatEPut of Trenwick underwritten by European Re in 2000 entitles Trenwick to sell 55 million shares to European Re, but European Re failed to meet its obligations and bought only 40 million.

¹⁰ In 1997, Horace Mann was upgraded by A.M. Best from A to A* due to the completion of its three-year, US\$100 million CatEPut with Centre Reinsurance Ltd.

¹¹ Jaimungal and Wang (2006), Lin et al. (2009), and Chang et al. (2011) adopt the Vasicek's interest rate model.

ties of the (re)insurer are the value of all future claims related to the outstanding policies. The change in liabilities is assumed to be stochastic and consists of two components. The first one reflects the normal variation in liabilities, including the effects of interest rate changes and other day-to-day small shocks, and is modeled as a continuous diffusion process. The second one reflects the catastrophe risk that the (re)insurer faces large jumps in liabilities and is assumed to be governed by a compound Poisson process. Therefore, the liability dynamics of an insurer ($L_{I,t}$) and reinsurer ($L_{R,t}$) under the risk-neutral measure \mathbb{Q} can be described as follows (at time t)¹²:

$$\frac{dL_{x,t}}{L_{x,t}} = (r_t - \lambda \bar{\mu}_x)dt + \sigma_{L,x} d\tilde{W}_{x,t} + Y_{x,t} dN_t, \quad x = I \text{ or } R, \quad (3)$$

where $\sigma_{L,x}$ is the total volatility of x -company's liabilities and $\tilde{W}_{x,t}$ is a Wiener process under \mathbb{Q} .¹³ We denote the correlation coefficient of $\tilde{W}_{x,t}$ and $Z_{r,t}$ by $\tilde{\rho}_{x,r}$. We assume the Wiener process driving the liabilities to be correlated with the risk-free rate, believing that the insurer's liabilities are in general highly interest-rate sensitive. The insurer's liabilities are typically valued by the present value of future claims related to the outstanding policies. The value of the insurer's liabilities fluctuates due to the variation of interest rates, insurance inflation, and the stochastic occurrence of claims. See [Staking and Babbal \(1995\)](#) and [Kuo et al. \(2003\)](#), among others, for empirical evidence.

The last term in (3) illustrates the catastrophe risk. Here, $Y_{x,t}$ is a sequence of independent and identically-distributed positive random variables describing the percentage change in liabilities due to catastrophes, and N_t is a Poisson process with intensity λ and is independent of other variables. We assume that $\ln(Y_{x,t})$ has a normal distribution with mean μ_x and standard deviation σ_x , which implies that the expected jump size for x -company is $\bar{\mu}_x = e^{\mu_x + \frac{1}{2}\sigma_x^2}$. This assumption ensures that a catastrophe always raises x -company's liabilities even though the magnitude is random. Here, we assume the insurer and reinsurer share the same catastrophe intensity, but endure different impacts. The correlation coefficient of $\ln(Y_{I,t})$ and $\ln(Y_{R,t})$ is ρ_Y . Due to jumps, the liability process is right continuous.

2.4. Insurer's share price

Consider that an insurer, with m_1 shares outstanding, intends to hedge its catastrophe risk by purchasing CatEPuts from a reinsurer. Here, $S_{I,t}$ denotes the insurer's share price at time t . The share price can be valued as follows:

$$S_{I,t} = \max \left\{ \frac{A_{I,t} - L_{I,t}}{m_1}, 0 \right\}. \quad (4)$$

2.5. Exercise style

We assume CatEPuts can be exercised on a periodic basis (weekly, monthly, etc.) to more closely reflect their American-style feature in practice and write $T_1, \dots, T_n \equiv T$. This study assumes that the CatEPut has 3 years to maturity ($T = 3$) and can be exercised at the end of every month ($n = 36$). However, the literature treats

¹² For the risk-neutral pricing measure, we follow [Merton \(1976\)](#) and [Cummins \(1988\)](#) by assuming that the overall economy is only marginally influenced by localized catastrophes. This assumption implies that $Y_{x,t}$ and N_t pertain to idiosyncratic shocks to the insurer's liabilities and have zero risk premium. See also [Naik and Lee \(1990\)](#) and [Shimko \(1992\)](#).

¹³ For pricing the contingent capital contracts in our structural model, the underlying liability is not really traded, but is typically assumed to be perfectly hedged or replicated on a sufficient liquid and transparent market. This assumption probably can be justified by the fact that there are highly active and transparent reinsurance markets and emerging insurance-linked securities markets.

CatEPuts as European-style options to derive the closed-form solution.¹⁴

2.6. Probability of default

In order to define the default event, we apply the first passage model proposed by [Black and Cox \(1976\)](#) through monthly examinations. In other words, the default occurs at the first time τ_x , such that x -company's assets are less than its liabilities:

$$\tau_x = \inf \{ t = T_1, \dots, T_n | A_{x,t} \leq L_{x,t} \}, \quad x = I \text{ or } R. \quad (5)$$

Therefore, the probability of default (PD) for the (re)insurer can be written as:

$$PD_x = \Pr(\min_{0 < t \leq T} \{ A_{x,t} - L_{x,t} \} \leq 0), \quad x = I \text{ or } R. \quad (6)$$

3. Valuation model

A CatEPut is an option in which the buyer has the right to issue a certain amount of its shares (denoted as m_2) at a specified price (denoted as K) only if the accumulated catastrophic losses exceed a predetermined amount (denoted as \mathcal{L}). In other words, this is a double trigger put option. The buyer/insurer can issue new shares of its stock to the seller/reinsurer at K per share if (i) the insurer's share price is less than the strike price and (ii) the catastrophic trigger condition is matched.

3.1. Catastrophic loss

CatEPuts become exercisable when the accumulated catastrophic losses faced by the insurer at maturity date T (denoted as $L_{I,T}^C$) are larger than a predetermined amount \mathcal{L} .¹⁵ This study allows the contract to be exercisable on a monthly basis and examines the accumulated catastrophic losses $L_{I,t}^C$ at the same time point accordingly. The catastrophic losses during the period of T_{i-1} to T_i can be described as total liabilities minus non-catastrophic liabilities:

$$L_{I,T_i}^C - L_{I,T_{i-1}}^C = L_{I,T_i} - L_{I,T_{i-1}} \times \exp \left\{ - \sum_{j=1+N_{T_{i-1}}}^{N_{T_i}} \ln(1 + Y_{I,j}) \right\}, \quad (7)$$

since

$$L_{I,T_i} = L_{I,T_{i-1}} \exp \left[\int_{T_{i-1}}^{T_i} r(u)du - \left(\frac{1}{2} \sigma_{L,I}^2 + \lambda \mu_x \right) (T_i - T_{i-1}) + \sigma_{L,I} (\tilde{W}_{I,T_i} - \tilde{W}_{I,T_{i-1}}) + \sum_{j=1+N_{T_{i-1}}}^{N_{T_i}} \ln(1 + Y_{I,j}) \right].$$

The last term in (7) represents total liabilities, excluding the catastrophic losses during T_{i-1} to T_i . If there is no catastrophe event ($N_{T_i} = N_{T_{i-1}}$), then the right-hand side in (7) is zero, which means no catastrophic losses during the period.

3.2. Payoff of contingent capital

Consider that the CatEPuts allow the insurer to issue m_2 shares to cover potential losses and the payoffs of CatEPuts without counterparty risk, PO_{T_i} , can be described as:

$$\begin{cases} m_2 (K - S_{I,T_i}^*), & \text{if } L_{I,T_i}^C \geq \mathcal{L} \text{ and } S_{I,T_i}^* < K, \\ 0, & \text{otherwise,} \end{cases} \quad (8)$$

¹⁴ For example, [Cox et al. \(2004\)](#), [Jaimungal and Wang \(2006\)](#), [Lin et al. \(2009\)](#), and [Chang et al. \(2011\)](#).

¹⁵ See also [Cox et al. \(2004\)](#), who define their catastrophic trigger as the number of a specified catastrophic event.

where $S_{i,t}^*$, the post-exercise share price, is defined by $(A_{i,t} - L_{i,t} + m_2 - K)/(m_1 + m_2)$.¹⁶ Note that all previous studies, such as Cox et al. (2004), Jaimungal and Wang (2006), and Chang et al. (2011), define the CatEput payoff using the pre-exercise share price, and their modeling implicitly assumes that insurance companies are selling their existing treasury shares to the CatEput buyer. However, in the real world insurance companies in fact issue new shares rather than sell their existing treasury shares to the CatEput buyer.¹⁷

Payoffs with counterparty risk, $PO_{T_i}^{CR}$, can be expressed as:

$$\begin{cases} m_2(K - S_{i,T_i}^*), & \text{if } L_{i,T_i}^C \geq \mathcal{L}, S_{i,T_i}^* < K, \text{ and } A_{R,T_i} - L_{R,T_i} > m_2(K - S_{i,T_i}^*), \\ \alpha_{T_i}(A_{R,T_i} - L_{R,T_i}), & \text{if } L_{i,T_i}^C \geq \mathcal{L}, S_{i,T_i}^* < K, \text{ and } A_{R,T_i} - L_{R,T_i} \leq m_2(K - S_{i,T_i}^*), \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

where $\alpha_t \equiv \frac{m_2(K - S_{i,t}^*)}{m_2(K - S_{i,t}^*) + L_{R,t}} \in (0, 1)$. The first line in (9) shows the CatEput's payoff when the reinsurer has enough net worth to meet the claim. The second line shows the payoff when the reinsurer does not have enough to meet the claim and the CatEput holders have to share the remaining assets with other liability claimants of the reinsurer on a *pro rata* basis.

3.3. Price of contingent capital

According to the payoff structures, the catastrophic loss process, and the dynamics for the (re)insurer's assets and liabilities specified above, the price of the contingent capital or the rate on line can be valued as follows:

$$P = \frac{1}{m_2K} \times E^Q \left[e^{-\int_0^{T_{\wedge\tau}} r(u)du} \times PO_{T_{\wedge\tau}}^* \right], \quad (10)$$

where $E^Q[\cdot]$ denotes expectations taken on the issuing date under a risk-neutral pricing measure, $PO_{T_{\wedge\tau}}^*$ is $PO_{T_{\wedge\tau}}$ for CSN and $PO_{T_{\wedge\tau}}^{CR}$ for CatEput, and τ denotes the first exercisable time defined as:

$$\tau = \inf \left\{ t = T_1, \dots, T_n \mid L_{i,t}^C \geq \mathcal{L} \text{ and } S_{i,t}^* < K \right\}. \quad (11)$$

Note that the total contingent capital brought in, m_2K , is fixed in spite of the exercise timing. It is composed of the put payoff, $m_2(K - S_{i,\tau}^*)$, and the new capital infusion, $m_2S_{i,\tau}^*$. Without further complicating our model, we assume the buyer prefers receiving the fixed contingent capital early rather than late. Alternatively, one can consider the strike price K corresponding to the critical level of regulatory capital that the buyer must maintain.

3.4. Price endogeneity

The price of a CatEput obviously depends on the initial asset values of the insurer and the reinsurer. When an insurer purchases CatEputs from the reinsurer, the insurer's asset value will decrease. This reduction leads to a higher exercise probability of the CatEput and increases the put's value. The purchase, however, raises the reinsurer's asset value and leads to a lower counterparty risk, therefore increasing the put's value. It follows that considering price endogeneity raises the CatEput price.¹⁸ In order to deal with this endogeneity, we propose a dynamic model to obtain the equilibrium price of a CatEput. The algorithm of deriving the price is as follows.

Algorithm 1.

-
- Step 1.** For $i = 0$, given the initial asset values $A_{i,0}^{(0)}$ and $A_{R,0}^{(0)}$, calculate the initial CatEput price $P^{(0)}$.
 - Step 2.** For $i = i + 1$, set $A_{i,0}^{(i)} = A_{i,0}^{(0)} - m_2KP^{(i-1)}$ and $A_{R,0}^{(i)} = A_{R,0}^{(0)} + m_2KP^{(i-1)}$.
 - Step 3.** Calculate the updated CatEput price $P^{(i)}$.
 - Step 4.** If $|P^{(i)} - P^{(i-1)}| > 10^{-6}$, then repeat **Step 2**;
If $|P^{(i)} - P^{(i-1)}| \leq 10^{-6}$, then we define the equilibrium price $P^* = P^{(i)}$.
-

Note that the equilibrium price does exist, because $P^{(i)}$ is an increasing function in i and bounded above by $A_{i,0}^{(0)}$.

4. Numerical analysis

This section estimates the default probabilities and CatEput prices using the Monte Carlo method with 250,000 paths. This section further presents parameters and their values and demonstrates how price endogeneity and counterparty risk affect CatEput prices. We provide the details of simulation procedures in Appendix A.

4.1. Parameters

As a reference point for the numerical results, Table 1 presents a base set of parameter values. The initial spot interest rate is set at 2%. The steady long-run rate is 5%, the magnitude of the mean-reverting force is 20%, the volatility of the interest rate is 3%, and the coefficient of correlation for random shocks of asset values (liabilities) and the interest rate, $\rho_{x,r}$ ($\hat{\rho}_{x,r}$), is set to be -0.5 . These interest rate parameter values are all within the range typically used in the previous literature such as Duan and Simonato (2002) and Jaimungal and Wang (2006). Next, this study assumes that the coefficient of correlation for random shocks of the asset values (liabilities) between the insurer and reinsurer, ρ_A (ρ_L), is 0.5, and the coefficient of correlation for the logarithm of catastrophe losses of the insurer and reinsurer, ρ_Y , is 0.5.

In order to investigate the significance of the counterparty risk, we consider low-risk and high-risk profiles for both the insurer and reinsurer in the sense of PD. The low-risk (re)insurer's rating is Moody's-A and S&P-A, and the high-risk (re)insurer's rating is Moody's-BB and S&P-Ba. We assume that the asset/liability ratios for the low-risk and high-risk companies are 1.3 and 1.2, respectively. The total volatilities of asset returns and liabilities are set to be 5% and 2%, respectively. These parameter values are from Cummins (1988), Duan and Yu (2005), and Pennacchi (2010).

When choosing the remaining parameter values, we follow Härdle and Cabrera (2010) to ensure that our simulated three-year PDs are in the range of those reported by Moody's and S&P as shown in Table 2. This study considers two cases of catastrophe intensity ($\lambda = 0.1$ and 0.25) and further assumes the low intensity comes with a higher impact on expected catastrophe losses, and vice versa. For the low-risk company, we set the expected jump size $\bar{\mu}_x = 4\%$ and 3% for the low- and high-intensity cases, respectively. For the high-risk company, we set the expected jump size $\bar{\mu}_x = 9\%$ and 6% for the low- and high-intensity cases, respectively. The standard deviation of the logarithm of catastrophe losses is 20% for all cases. As a result, the three-year PDs of the low-risk and high-risk companies are about 0.3% and 6%, respectively.

Deviations from the base values provide insights into how changes in the characteristics of the insurer, reinsurer, and

¹⁶ Note that when $S_{i,t}$ denotes the surplus notes' price and K denotes the Treasury note, it is the same payoffs of CSN as discussed in Lin et al. (2009).

¹⁷ The authors thank the referee for pointing out this issue.

¹⁸ Note that CSNs and CAT bonds do not have the problem of price endogeneity.

Table 1
Parameters, definitions, and base values.

	Values	
<i>Interest rate parameters</i>		
r_t	Instantaneous interest rate	$r_0 = 2\%$
κ	Magnitude of mean-reverting force	20%
μ_r	Long-run mean of the interest rate	5%
σ_r	Volatility of the interest rate	3%
<i>Asset and liability parameters</i>		
$L_{i,t}$	Insurer's liabilities	1
$A_{i,t}$	Insurer's assets	$A_{i,0}/L_{i,0} = 1.2$ or 1.3
$A_{R,t}$	Reinsurer's assets	$A_{R,0}/A_{i,0} = 1$
$L_{R,t}$	Reinsurer's liabilities	$A_{R,0}/L_{R,0} = 1.2$ or 1.3
$\sigma_{A,x}$	Total volatility of (re)insurer's asset return	5%
$\sigma_{L,x}$	Total volatility of (re)insurer's liability	2%
<i>Catastrophe loss parameters</i>		
λ	Catastrophe intensity	0.1 or 0.25
$\bar{\mu}_x$	Expected jump size for the (re)insurer	9% or 4% if $\lambda = 0.1$, 6% or 3% if $\lambda = 0.25$.
σ_x	Standard deviation of the logarithm of catastrophe losses	20%
<i>Correlations</i>		
ρ_A	Correlation coefficient of credit shocks between the insurer and the reinsurer	0.5
ρ_L	Correlation coefficient of pure liability risk between the insurer and the reinsurer	0.5
ρ_V	Correlation coefficient of the logarithms of catastrophe losses between the insurer and the reinsurer	0.5
$\rho_{x,r}$	Correlation coefficient between the interest rate and assets	-0.5
$\bar{\rho}_{x,r}$	Correlation coefficient between the interest rate and liabilities	-0.5
<i>Other parameters</i>		
T	Time to maturity	3
T_i	Examination dates, $i = 1, \dots, n$.	$T_i = i/12, n = 36$.
τ_x	First passage time of x -company's default	
τ	First exercisable time	
$S_{i,0}$	Insurer's initial share price $\equiv (A_{i,0} - L_{i,0})/m_1$	
K	Strike price	$0.8S_{i,0}$ (out of the money)
\mathcal{L}	Trigger level	$0.1L_{i,0}$
m_1	Shares outstanding	1
m_2	Underlying shares of CatEPut as % of m_1	20% and 50%

Table 2

Three-year probability of default (%) rating agencies vs. simulation values. Simulation values are calculated assuming the first passage model proposed by Black and Cox (1976) with monthly examinations, the volatility of asset $\sigma_{A,x} = 5\%$, the volatility of liability $\sigma_{L,x} = 2\%$, and $\frac{A_{x,0}}{L_{x,0}} = 1.2(1.3)$ for the high (low)-risk company. Case $\lambda = 0.1$ exhibits a scenario of low intensity and large expected losses, where $\bar{\mu}_x = 9\%(4\%)$ for the high (low)-risk company. Case $\lambda = 0.25$ exhibits a scenario of high intensity and small expected losses, where $\bar{\mu}_x = 6\%(3\%)$ for the high (low)-risk company. All values are estimated using the Monte Carlo simulation with 250,000 runs.

Risk profile	Moody's rating		S&P rating		Simulation	
	A	BB	A	Ba	$\lambda = 0.1$	$\lambda = 0.25$
Low-risk	0.25		0.31		0.28	0.31
High-risk		6.20		5.61	5.89	6.18

catastrophe events affect the CatEPut price. We consider the relative ratio of the asset size of the reinsurer/insurer to be 1. For simplicity, we let the insurer's shares outstanding, m_1 , be 1, and the underlying shares of the put, m_2 , are set at 20% or 50% of m_1 . The strike price K is set to be out-of-the-money and at $0.8S_{i,0}$. The catastrophe trigger level is set at $0.1L_{i,0}$.

4.2. CatEPut price with price endogeneity

Table 3 reports CatEPut prices in basis points under four scenarios: two risk profiles for the buyer across two levels of catastrophe intensity. In this table we do not consider the counterparty risk, and so the seller's risk profile is irrelevant. For each scenario, we further report CatEPut prices for m_2 at 20% and 50%. CatEPut price $P^{(0)}$ stands for the price without considering the purchasing cost, and it is decreasing in m_2 , because a larger m_2 implies a higher post-exercise share price ($S_{i,t}^*$), and therefore a smaller payoff as indicated in (8). For the high-risk buyer, the prices are 407.35, 334.76, 553.26, and 450.17 basis points for cases $(\lambda, m_2) = (0.1, 0.2)$, $(0.1, 0.5)$, $(0.25, 0.2)$, and $(0.25, 0.5)$, respectively; and the corresponding prices for the low-risk buyer are 14.54, 11.65, 30.79, and 24.70 basis points, respectively.

When incorporating the purchasing cost, prices are endogenously determined and will rise to $P^{(*)}$. We demonstrate the convergence of CatEPut prices, and all prices converge within 4 steps of iterations. The difference between $P^{(*)}$ and $P^{(0)}$ is exhibited at the bottom line of the table, indicating that the higher the ratio of the insurer's shares covered by the CatEPut (m_2) is, the stronger the impact of price endogeneity. The price underestimation due to the endogeneity is significant and ranges from 5.18 to 18.11 basis points for the high-risk buyer; on the contrary, it is insignificant for the low-risk buyer, because the purchasing cost of the CatEPut is too small in this case.

4.3. CatEPut price with counterparty risk

Table 4 shows the effect of counterparty risk on the CatEPut price under the consideration of price endogeneity. Panel (a) reports the prices without counterparty risk, and panels (b) and (c) report the prices with counterparty risk. We consider low- and high-risk profiles for the CatEPut seller (i.e. reinsurer), and the seller's asset scale to the buyer's, $\frac{A_{R,0}}{A_{i,0}}$, is either 5 or 1.

We set the case without counterparty risk as the benchmark and define the difference from the benchmark price as the counterparty risk premium (CRP). Since counterparty risk decreases the value of the CatEPut, prices in panels (b) and (c) are lower than their corresponding prices in (a). Moreover, CatEPut prices for the high-risk seller in panel (c) are lower than their corresponding values in panel (b) with low counterparty risk. A higher ratio of shares covered by the put, m_2 , means the more sensitive the price is to counterparty risk. Hence, counterparty risk lowers the price more for the case of $m_2 = 50\%$ than it does for $m_2 = 20\%$.

For the high-risk seller, the counterparty risk becomes more substantial. It may decrease the price by 80.80–166.01 basis points for the high-risk buyer and 9.07–20.98 basis points for the low-risk buyer. For the low-risk seller, the counterparty risk is relatively small, but significant when the seller's asset scale is close to the buyer's, and it still can decrease the price by more than 11 basis points for some cases. In addition, we note that prices increase with the asset scale ratio when it rises from 1 to 5, but this effect is not as significant as that due to the change in the seller's risk profile.

Since counterparty risk and the price endogeneity affect the prices in different directions, the joint effect is undetermined. Fig. 1 exhibits the CatEPut prices with and without considering these two effects and demonstrates the relation between CatEPut prices and the coverage ratio. For a high-risk seller, the CRP dominates the price endogeneity effect and the CatEPut price is much lower than that without considering counterparty risk and the price endogeneity. For a low-risk seller, the price endogeneity becomes more substantial and may offset the counterparty risk, and consequently the CatEPut price is insignificantly different from

Table 3
CatEPut prices in basis points with vs. without price endogeneity. All values are calculated assuming option term $T = 3$, monthly exercisable, without counterparty risk, strike price $K = 0.8 S_{I,0}$, trigger level $\mathcal{L} = 0.1 L_{I,0}$, shares outstanding $m_1 = 1$, the volatility of asset $\sigma_{A,t} = 5\%$, the volatility of liability $\sigma_{L,t} = 2\%$, and $\frac{A_{I,0}}{L_{I,0}} = 1.2(1.3)$ for the high (low)-risk insurer. Case $\lambda = 0.1$ exhibits a scenario of low intensity and large expected losses, where $\bar{\mu}_t^* = 9\%(4\%)$ for the high (low)-risk insurer. Case $\lambda = 0.25$ exhibits a scenario of high intensity and small expected losses, where $\bar{\mu}_t^* = 6\%(3\%)$ for the high (low)-risk insurer. All values are estimated using the Monte Carlo simulation with 250,000 runs, and standard errors are reported in brackets.

$m_2 =$	High-risk buyer				Low-risk buyer			
	$\lambda = 0.1$		$\lambda = 0.25$		$\lambda = 0.1$		$\lambda = 0.25$	
	20%	50%	20%	50%	20%	50%	20%	50%
$P^{(0)}$	407.35 [3.097]	334.76 [2.593]	553.26 [3.475]	450.17 [2.867]	14.54 [0.510]	11.65 [0.410]	30.79 [0.718]	24.70 [0.578]
$P^{(1)}$	412.47	343.72	563.60	467.59	14.55	11.67	30.84	24.79
$P^{(2)}$	412.53	343.97	563.74	468.27	14.55	11.67	30.84	24.79
$P^{(3)}$	412.53	343.98	563.74	468.28				
$P^{(4)}$		343.98		468.28				
$P^{(*)}$	412.53 [3.123]	343.98 [2.641]	563.74 [3.518]	468.28 [2.945]	14.55 [0.511]	11.67 [0.410]	30.84 [0.718]	24.79 [0.579]
$P^{(*)} - P^{(0)}$	5.18 [0.078]	9.22 [0.096]	10.48 [0.124]	18.11 [0.157]	0.01 [0.002]	0.02 [0.002]	0.05 [0.005]	0.09 [0.005]

Table 4
CatEPut prices in basis points with vs. without counterparty risk. All values are calculated assuming option term $T = 3$, monthly exercisable, strike price $K = 0.8 S_{I,0}$, trigger level $\mathcal{L} = 0.1 L_{I,0}$, shares outstanding $m_1 = 1$, the volatility of asset $\sigma_{A,t} = 5\%$, the volatility of liability $\sigma_{L,t} = 2\%$, and $\frac{A_{I,0}}{L_{I,0}} = 1.2(1.3)$ for the high (low)-risk company. Case $\lambda = 0.1$ exhibits a scenario of low intensity and large expected losses, where $\bar{\mu}_t^* = 9\%(4\%)$ for the high (low)-risk company. Case $\lambda = 0.25$ exhibits a scenario of high intensity and small expected losses, where $\bar{\mu}_t^* = 6\%(3\%)$ for the high (low)-risk company. CRP denotes the counterparty risk premium. All values are estimated using the Monte Carlo simulation with 250,000 runs, and standard errors are reported in brackets.

$m_2 =$	High-risk buyer				Low-risk buyer			
	$\lambda = 0.1$		$\lambda = 0.25$		$\lambda = 0.1$		$\lambda = 0.25$	
	20%	50%	20%	50%	20%	50%	20%	50%
(a) Without counterparty risk	412.53	343.98	563.74	468.28	14.55	11.67	30.84	24.79
(b) With counterparty risk: low-risk seller								
$\frac{A_{R,0}}{A_{I,0}} = 5$	410.36	341.20	561.38	465.27	13.82	10.95	29.91	23.73
CRP	2.17 [0.28]	2.78 [0.31]	2.36 [0.29]	3.01 [0.31]	0.74 [0.14]	0.72 [0.13]	0.93 [0.16]	1.06 [0.16]
$\frac{A_{R,0}}{A_{I,0}} = 1$	407.24	332.43	557.37	455.35	13.39	10.23	28.84	21.70
CRP	5.29 [0.44]	11.55 [0.63]	6.37 [0.48]	12.93 [0.64]	1.16 [0.18]	1.44 [0.18]	2.00 [0.23]	3.09 [0.26]
(c) With counterparty risk: high-risk seller								
$\frac{A_{R,0}}{A_{I,0}} = 5$	331.73	260.02	480.61	380.71	3.62	2.60	13.08	9.61
CRP	80.80 [1.60]	83.96 [1.54]	83.13 [1.57]	87.57 [1.50]	10.93 [0.46]	9.07 [0.38]	17.76 [0.58]	15.17 [0.48]
$\frac{A_{R,0}}{A_{I,0}} = 1$	287.57	201.63	424.82	302.27	2.63	1.62	9.86	5.91
CRP	126.96 [1.99]	142.34 [1.96]	138.92 [2.03]	166.01 [2.03]	11.92 [0.48]	10.05 [0.39]	20.98 [0.63]	18.88 [0.53]

that without considering counterparty risk and the price endogeneity.

5. Further discussions

5.1. CatEPut transaction and credit rating

In this section we examine how CatEPuts affect the insurer's default risk. Since CatEPuts ensure that the buyer can issue new shares when the put is exercised, the total change in PD (ΔPD_I) includes a pure put option effect (ΔPD_I^p) and a new equity effect (ΔPD_I^n):

$$\Delta PD_I = \Delta PD_I^p + \Delta PD_I^n \tag{12}$$

By looking only into the pure put option effect, one cannot tell whether its change is positive or negative. CatEPuts cover part of the insurer's catastrophic losses and decrease its PD, but purchasing

CatEPuts decreases the insurer's future cash flows and increases its PD.

In order to precisely measure the change in PD, we further decompose the pure put option effect into three parts: a reduction of PD due to the payoff of the CatEPut (PD_I^{po}), an increase in PD coming from counterparty risk (PD_I^{cr}), and another increase due to the expenses of acquiring the option (PD_I^{pe}). The decomposition is as follows:

$$\Delta PD_I^p = PD_I^{po} + PD_I^{cr} + PD_I^{pe}, \tag{13}$$

and

$$PD_I^{po} \equiv Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^0 + PO_{t \wedge T} - L_{I,t} \} \leq 0 \right) - Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^0 - L_{I,t} \} \leq 0 \right), \tag{14}$$

$$PD_I^{cr} \equiv Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^0 + PO_{t \wedge T}^{CR} - L_{I,t} \} \leq 0 \right) - Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^0 + PO_{t \wedge T} - L_{I,t} \} \leq 0 \right), \tag{15}$$

$$PD_I^{pe} \equiv Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^* + PO_{t \wedge T}^{CR} - L_{I,t} \} \leq 0 \right) - Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^0 + PO_{t \wedge T}^{CR} - L_{I,t} \} \leq 0 \right), \tag{16}$$

where $A_{I,t}^0$ and $A_{I,t}^*$ denote the insurer's asset value at time t with initial value $A_{I,0}$ and $(A_{I,0} - P^*)$, respectively. Terms PO_t and PO_t^{CR} represent the corresponding payoffs brought in by the put as defined in (8) and (9). It is worth noting that PD_I^{po} is positive, PD_I^{cr} and PD_I^{pe} are negative, and the sign of ΔPD_I^p is undetermined.¹⁹

The new equity effect on PD is surely negative since it brings in new capital. It can be expressed as:

$$\Delta PD_I^n \equiv Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^* + CI_{t \wedge T} - L_{I,t} \} \leq 0 \right) - Pr \left(\min_{0 < t \leq T} \{ A_{I,t}^* + PO_{t \wedge T}^{CR} - L_{I,t} \} \leq 0 \right),$$

where $CI_t \equiv PO_t^{CR} + m_2 S_{I,t}^* \mathbf{1}_{\{t \leq T\}}$ is the total capital infusion at the trigger point. Note that the new equity effect is defined as improve-

¹⁹ Similar arguments can be applied to the seller/reinsurer to define its change in PD as $\Delta PD_R = PD_R^{po} + PD_R^{pe}$. We decompose the reinsurer's change in PD (ΔPD_R) into two parts: the increase in PD due to the loss coverage of the insurance put (PD_R^{po}) and the reduction in PD due to the revenue for acquiring the put (PD_R^{pe}). However, this study's discussion focuses on the buyer.

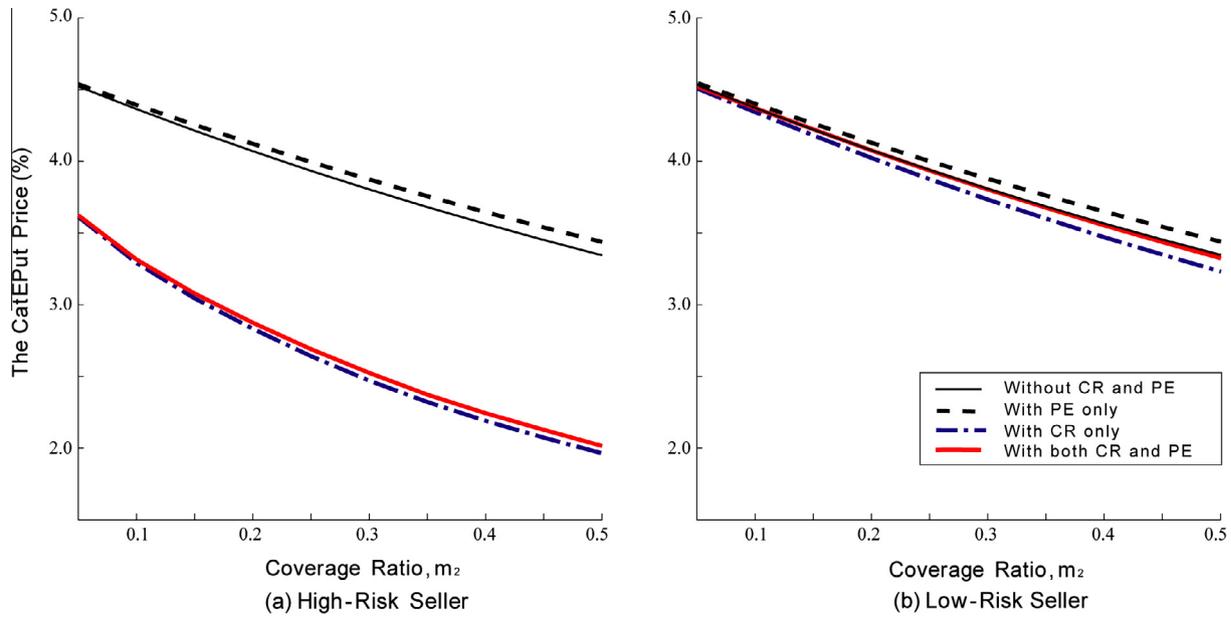


Fig. 1. The joint effect of counterparty risk and price endogeneity. All values are estimated using the Monte Carlo simulation with 250,000 runs and assuming option term $T = 3$, monthly exercisable, a high-risk buyer, strike price $K = 0.8S_{t,0}$, the trigger level $\mathcal{L} = 0.1L_{t,0}$, and the catastrophe intensity $\lambda = 0.1$. Notations CR and PE denote the counterparty risk effect and price endogeneity effect, respectively.

ments in PD contributed by the capital infusion beyond the put payoff effect.

Table 5 demonstrates the change in PD before (PD_i^0) and after (PD_i^1) the insurer buys the CatEPut. First, we note that the total effect of changes in PDs are all negative without exception. For the high-risk buyer, we observe a significant credit enhancement with a reduction in PD in the range of 0.76–2.48%; however, for the low-risk buyer the changes in panels (c) and (d) are all close to zero, implying a limited credit enhancement. Next, we further decompose the credit enhancement into the pure put option effect and

the new equity effect. Our results indicate that both effects alone can lower the insurer's PD.

If the counterparty risk and price endogeneity are ignored, then the credit enhancement of the high-risk buyer will further grow to 1.05–2.98%. Particularly for the cases of (0.10%,50%) and (0.25%,50%) in panel (b), the credit enhancement effect will be significantly overestimated by 0.77% and 0.94%, respectively. These findings further support our concerns that counterparty risk and price endogeneity should be included in the valuation of contingent capital.

Table 5

Change in insurer's probability of default with CatEPuts (%). Simulation values are calculated assuming the first passage model proposed by Black and Cox (1976) on a monthly basis, option term $T = 3$, strike price $K = 0.8S_{t,0}$, trigger level $\mathcal{L} = 0.1L_{t,0}$, shares outstanding $m_1 = 1$, the volatility of asset $\sigma_{A,x} = 5\%$, the volatility of liability $\sigma_{L,x} = 2\%$, and $\frac{A_{t,0}}{L_{t,0}} = 1.2(1.3)$ for the high (low)-risk company. Case $\lambda = 0.1$ exhibits a scenario of low intensity and large expected losses, where $\bar{\mu}_x = 9\%(4\%)$ for the high (low)-risk company. Case $\lambda = 0.25$ exhibits a scenario of high intensity and small expected losses, where $\bar{\mu}_x = 6\%(3\%)$ for the high (low)-risk company. ΔPD_i^{p0} , ΔPD_i^{cr} , ΔPD_i^{pe} , ΔPD_i^n , and ΔPD_i denote the payoff effect, the counterparty risk effect, the price endogeneity effect, the new equity effect, and the total effect on probability of default, respectively. PD_i^0 and PD_i^1 denote the default probability before and after the CatEPut transaction, respectively. All values are estimated using the Monte Carlo simulation with 250,000 runs.

(λ, m_2)	Pure put option effect			+ New equity effect	= Total effect		
	ΔPD_i^{p0}	$+\Delta PD_i^{cr} +$	ΔPD_i^{pe}	ΔPD_i^n	ΔPD_i	$= PD_i^1 -$	PD_i^0
<i>(a) High-risk buyer, low-risk seller</i>							
(0.10,20%)	-0.71	0.00	0.13	-0.34	-0.91	4.97	5.89
(0.10,50%)	-1.38	0.03	0.23	-0.76	-1.88	4.00	5.89
(0.25,20%)	-0.98	0.01	0.17	-0.48	-1.29	4.89	6.18
(0.25,50%)	-1.81	0.04	0.29	-1.00	-2.48	3.70	6.18
<i>(b) High-risk buyer, high-risk seller</i>							
(0.10,20%)	-0.71	0.21	0.09	-0.35	-0.76	5.12	5.89
(0.10,50%)	-1.38	0.64	0.13	-0.86	-1.47	4.42	5.89
(0.25,20%)	-0.98	0.28	0.12	-0.50	-1.09	5.09	6.18
(0.25,50%)	-1.81	0.75	0.19	-1.16	-2.04	4.14	6.18
<i>(c) Low-risk buyer, low-risk seller</i>							
(0.10,20%)	-0.02	0.00	0.00	-0.00	-0.02	0.26	0.28
(0.10,50%)	-0.02	0.01	0.00	-0.01	-0.02	0.25	0.28
(0.25,20%)	-0.03	0.00	0.00	-0.01	-0.04	0.27	0.31
(0.25,50%)	-0.05	0.01	0.00	-0.02	-0.05	0.26	0.31
<i>(d) Low-risk buyer, high-risk seller</i>							
(0.10,20%)	-0.02	0.02	0.00	-0.01	-0.01	0.27	0.28
(0.10,50%)	-0.02	0.02	0.00	-0.02	-0.02	0.26	0.28
(0.25,20%)	-0.03	0.03	0.00	-0.02	-0.02	0.29	0.31
(0.25,50%)	-0.05	0.04	0.00	-0.04	-0.04	0.27	0.31

Table 6
 CatEPut prices in basis points correlation of catastrophe risk. All values are calculated assuming option term $T = 3$, monthly exercisable, strike price $K = 0.8S_{t,0}$, trigger level $\mathcal{L} = 0.1L_{t,0}$, shares outstanding $m_1 = 1$, the volatility of asset $\sigma_{A,x} = 5\%$, the volatility of liability $\sigma_{L,x} = 2\%$, and $\frac{\lambda_{x,0}}{\lambda_{x,0}} = 1.2(1.3)$ for the high (low)-risk company. Case $\lambda = 0.1$ exhibits a scenario of low intensity and large expected losses, where $\bar{\mu}_x = 9\%(4\%)$ for the high (low)-risk company. Case $\lambda = 0.25$ exhibits a scenario of high intensity and small expected losses, where $\bar{\mu}_x = 6\%(3\%)$ for the high (low)-risk company. All values are estimated using the Monte Carlo simulation with 250,000 runs, and standard errors are reported in brackets.

$m_2 =$	High-risk buyer				Low-risk buyer			
	$\lambda = 0.1$		$\lambda = 0.25$		$\lambda = 0.1$		$\lambda = 0.25$	
	20%	50%	20%	50%	20%	50%	20%	50%
<i>(a) With counterparty risk: low-risk seller</i>								
$\rho_Y = 0$	407.42	332.27	557.57	455.76	13.40	10.28	29.01	21.87
$\rho_Y = 1$	407.28	331.97	557.52	454.41	13.36	10.08	28.80	21.52
difference	0.14	0.30	0.06	1.35	0.04	0.20	0.21	0.35
	[0.26]	[0.31]	[0.25]	[0.30]	[0.07]	[0.08]	[0.12]	[0.11]
<i>(b) With counterparty risk: high-risk seller</i>								
$\rho_Y = 0$	292.77	209.74	428.80	306.58	3.41	2.12	10.88	6.93
$\rho_Y = 1$	279.49	194.14	420.12	296.70	1.94	1.10	8.73	5.18
difference	13.27	15.60	8.68	9.87	1.47	1.02	2.15	1.75
	[1.14]	[0.88]	[1.19]	[0.97]	[0.16]	[0.11]	[0.23]	[0.17]

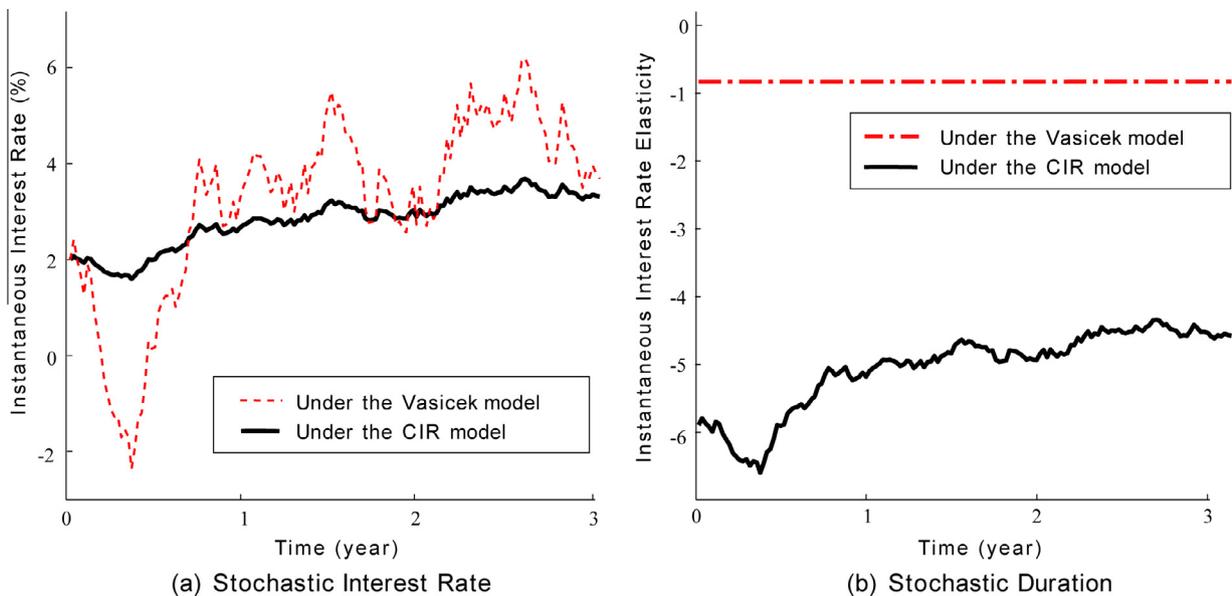


Fig. 2. Stochastic interest rate and stochastic duration. In this example, the initial instantaneous interest rate is $r_0 = 2\%$, magnitude of mean-reverting force $\kappa = 20\%$, long-run mean of interest rate $\mu_r = 5\%$, volatility of interest rate $\sigma_r = 3\%$, and correlation coefficient between interest rate and assets $\rho_{A,r} = -0.5$.

5.2. Correlation of catastrophe risk

Table 6 examines the correlation of catastrophe losses between the insurer and reinsurer in order to look further into the impact of counterparty risk. Intuitively, a higher correlation implies a higher default risk for the reinsurer since the reinsurer also suffers catastrophe losses when the insurer’s CatEPut is triggered. Table 6 demonstrates two extreme levels of correlation: uncorrelated ($\rho_Y = 0$) and perfectly correlated ($\rho_Y = 1$), for the low-risk seller and the high-risk seller. Consistent with the conjecture, CatEPut prices are negatively related to the correlation of catastrophe risk. The impact due to the catastrophe correlation risk is insignificant for the low-risk seller as in panel (a), but it is significant and could be as large as 15.60 basis points for the high-risk seller as in panel (b) under our reasonable scenarios. For example, for the case of the high-risk buyer, $\lambda = 0.1$, and the high-risk seller, the difference between $\rho_Y = 0$ and $\rho_Y = 1$ is 13.27 basis points when $m_2 = 20\%$ and is 15.60 basis points when $m_2 = 50\%$, both values are statistically significant.

5.3. Interest rate risk

There is no doubt that the interest rate is a major source of risk for both the insurer and reinsurer. In order to explicitly examine the interest rate risk exposure of the two, we apply the model in Duan et al. (1995) to rewrite the typical asset dynamics (2) and the liability dynamics (3) of x -company as follows²⁰:

$$\frac{dA_{x,t}}{A_{x,t}} = (r_t - \phi_{x,t}\kappa(\mu_r - r_t))dt + \phi_{x,t}dr_t + \psi_x dZ_{x,t}, \tag{17}$$

$$\frac{dL_{x,t}}{L_{x,t}} = \left(r_t - \tilde{\phi}_{x,t}\kappa(\mu_r - r_t) - \lambda e^{\mu_x + \frac{1}{2}\sigma_x^2} \right) dt + \tilde{\phi}_{x,t}dr_t + \tilde{\psi}_x d\tilde{Z}_{x,t} + Y_{x,t}dN_t, \tag{18}$$

where $\phi_{x,t}$ ($\tilde{\phi}_{x,t}$) is the instantaneous interest rate elasticity of x -company’s assets (liabilities); ψ_x is the volatility of the credit risk;

²⁰ The justification and details can be found in Appendix A.

Table 7

CatEPut prices in basis points interest rate risk. All values are calculated assuming option term $T = 3$, monthly exercisable, without counterparty risk, strike price $K = 0.8S_{t,0}$, trigger level $\mathcal{L} = 0.1L_{t,0}$, shares outstanding $m_1 = 1$, the volatility of the credit risk $\psi_{A,x} = 4.330\%$, the volatility of the pure liability risk $\psi_{L,x} = 1.732\%$, and $\frac{\lambda_{x,0}}{\bar{\mu}_x} = 1.2(1.3)$ for the high (low)-risk company. Case $\lambda = 0.1$ exhibits a scenario of low intensity and large expected losses, where $\bar{\mu}_x = 9\%(4\%)$ for the high (low)-risk company. Case $\lambda = 0.25$ exhibits a scenario of high intensity and small expected losses, where $\bar{\mu}_x = 6\%(3\%)$ for the high (low)-risk company. $\phi_{I,0}$ and $\tilde{\phi}_{I,0}$ denote the instantaneous interest rate elasticity of the insurer's assets and liabilities, respectively. All values are estimated using the Monte Carlo simulation with 250,000 runs, and standard errors are reported in brackets.

$m_2 =$	High-risk buyer				Low-risk buyer			
	$\lambda = 0.1$		$\lambda = 0.25$		$\lambda = 0.1$		$\lambda = 0.25$	
	20%	50%	20%	50%	20%	50%	20%	50%
(a) With interest rate risk ($\phi_{I,0}, \tilde{\phi}_{I,0}$) = (-5.892, -2.357)	412.53	343.98	563.74	468.28	14.55	11.67	30.84	24.79
(b) Without interest rate risk ($\phi_{I,0}, \tilde{\phi}_{I,0}$) = (0, 0)	424.44	355.58	581.64	486.69	15.62	12.55	34.26	27.57
(b)-(a)	11.91 [0.86]	11.60 [0.72]	17.90 [0.97]	18.41 [0.81]	1.07 [0.17]	0.88 [0.14]	3.42 [0.28]	2.78 [0.23]

$\tilde{\psi}_x$ is the volatility of the pure liability risk of x -company; $Z_{x,t}$ is a Wiener process under \mathbb{Q} that denotes the credit risk on the assets of x -company that is orthogonal to the interest rate risk; $\tilde{Z}_{x,t}$ is a Wiener process under \mathbb{Q} that summarizes all continuous shocks that are not related to the interest rate or asset risk of x -company, or in other words, $Z_{x,t}$, $\tilde{Z}_{x,t}$, and $Z_{r,t}$ are independent. The correlation coefficient of $Z_{I,t}$ and $Z_{R,t}$ is ρ_A , and the correlation coefficient of $\tilde{Z}_{I,t}$ and $\tilde{Z}_{R,t}$ is ρ_L .

Note that the instantaneous interest rate elasticity, $\phi_{x,t}$, in Duan et al. (1995) is a constant due to their Vasicek interest rate assumption. Under the CIR model, the elasticity depends on the interest rate level, $\phi_{x,t} = \frac{\sigma_{Ax}}{\sigma_r \sqrt{r_t}} \rho_{x,r}$, and captures the stochastic modified duration. Fig. 2 compares the Vasicek and the CIR interest rate models using an example. It shows the well-known property of the Vasicek model in which the model may generate a negative interest rate value. It then demonstrates that the instantaneous interest rate elasticity is stochastic for the CIR model, but constant for the Vasicek model.

In order to measure the interest rate risk, we consider the price difference from the interest rate-sensitive and -insensitive scenarios.²¹ Table 7 compares the insurer's interest rate risk and demonstrates how the interest rate risk affects the CatEPut prices. Panel (a) exhibits the case with initial elasticities $(\phi_{I,0}, \tilde{\phi}_{I,0}) = (-5.892, -2.357)$, which are derived from the base set of parameter values, and panel (b) exhibits the interest rate-insensitive case, $(\phi_{I,t}, \tilde{\phi}_{I,t}) = (0, 0)$, for all t . The numerical results show that the decrease due to the interest rate risk is economically and statistically significant in general, and the interest rate risk may decrease CatEPut prices by 11–18 basis points in some cases. In short, our model indeed provides a platform to evaluate the interest rate risk and to depict the stochastic modified duration. We also expect that the interest rate risk will be greater for contracts with longer maturity.

6. Summary remarks

This study has developed a dynamic structural framework to value contingent capital with counterparty risk and price endogeneity. Our analysis focuses on the CatEPut, which is the major and more complex instrument of contingent capital. The structural model allows the default risk to be determined endogenously, depending on the asset-liability structure of the insurer and the reinsurer. Our model also improves upon the literature by allowing

insurance companies to raise contingent capital through issuing new shares rather than selling their existing treasury shares to the CatEPut buyer, which is in accordance with the real world practices of CatEPut transactions. In addition, it provides a platform to evaluate critical parameters, such as interest rate risk and correlation risk, and to analyze how contingent capital affects the probability of default for the buyer and seller. These interesting issues cannot be addressed in the reduced-form literature.

Our results show that counterparty risk significantly decreases the CatEPut price and the extent of the decrease can easily exceed 80 basis points. The price endogeneity increases the CatEPut price; and the higher the price is, the stronger the impact. The increase in price due to price endogeneity can reach 5–18 basis points under some scenarios. These two factors are both substantial, but have not been considered in previous studies. Under reasonable scenarios, counterparty risk dominates the price endogeneity and CatEPut prices will be lower after considering these two factors. This study also improves upon the literature that has valued CatEPuts as a European-style contingent claim by allowing monthly exercises.

This study also finds that buying CatEPut can significantly decrease the PD for a high-risk insurer by 0.76–2.48% (no matter the risk-profile of its counterparty), but it may not significantly decrease the PD for a low-risk insurer under our reasonable scenarios. However, without taking the counterparty risk and price endogeneity into account, one may significantly overestimate the credit enhancement provided by the CatEPuts and underestimate the insurer's credit risk. These findings further support our concerns that counterparty risk and price endogeneity should be included in the valuation of contingent capital.

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Appendix A. Simulation procedures

We generate the interest rate process (1) by applying the Euler discretization at each T_i as

²¹ Duan et al. (1995) consider the difference in the interest rate elasticities of assets and liabilities.

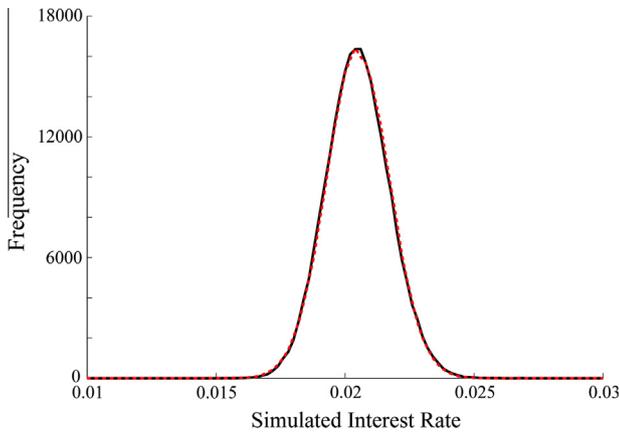


Fig. 3. Distributions of simulated interest rate. A comparison of exact distribution (solid) by generating the noncentral chi-square random numbers and the Euler approximation (dashed) by generating the normal random numbers as Eq. (A.1) for interest rate model (1) with parameters $r_0 = 2\%$, $\kappa = 0.2$, $\mu_r = 0.05$, $\sigma_r = 0.03$, $(T_i - T_{i-1}) = 1/12$, and 250,000 paths.

$$r_{T_i} = r_{T_{i-1}} + \kappa(\mu_r - r_{T_{i-1}})(T_i - T_{i-1}) + \sigma_r \sqrt{r_{T_{i-1}}^+} (Z_{r,T_i} - Z_{r,T_{i-1}}), \quad i = 1, 2, \dots, n, \tag{A.1}$$

where $Z_{r,s} - Z_{r,t}$ is normally distributed with mean zero and variance $s - t$, and $r_t^+ \equiv \max\{r_t, 0\}$ is the positive part of r_t .²² Comparing with the conditional density approach of Cox et al. (1985), the mean and the standard deviation of both approaches in our 250,000 simulations are less than 10^{-4} for all T_i .²³ In addition, the skewness and the kurtosis of both approaches are on average 0.015 and 0.033, respectively, and the whole distribution of both approaches are very closed as shown in Fig. 3. Therefore, we claim that the discretization errors in this study can be neglected due to the small time period $(T_i - T_{i-1}) = 1/12$.

After generating the paths of the interest rate, we then simulate the dynamics of assets and liabilities. Applying Ito's lemma, we can rewrite (2) and (3) into the following equations.

$$A_{x,T_i} = A_{x,T_{i-1}} \times \exp \left(\int_{T_{i-1}}^{T_i} r_s ds - \frac{1}{2} \sigma_{A,x}^2 (T_i - T_{i-1}) + \sigma_{A,x} (W_{x,T_i} - W_{x,T_{i-1}}) \right), \tag{A.2}$$

²² We take r_t^+ inside the square root, because the Euler discretization may generate a negative value of r_t . However, in our numerical analysis there is no negative r_t for all paths.

²³ Cox et al. (1985) show that the conditional interest rate under the CIR model (1) follows a non-central chi-square distribution. Given the initial value r_0 , the interest rate at the end of each period can be generated by:

$$r_{T_i} = \frac{\chi_v^2(\xi(T_{i-1}, T_i))}{2c(T_{i-1}, T_i)}, \quad i = 1, 2, \dots, n,$$

where $\chi_v^2(\xi)$ is a random number chosen from the non-central chi-square distribution with the degrees of freedom in v and the non-centrality ξ . In particular,

$$c(t, s) \equiv \frac{2\kappa}{\sigma_r^2(1 - e^{-\kappa(s-t)}), \quad \xi(t, s) \equiv 2c(t, s)r_t e^{-\kappa(s-t)}, \quad \text{and} \quad v \equiv \frac{4\kappa\mu_r}{\sigma_r^2}.$$

In this study we use the Matlab function "ncx2rnd" to generate the random numbers.

$$L_{x,T_i} = L_{x,T_{i-1}} \exp \left(\int_{T_{i-1}}^{T_i} r_s ds - \left(\lambda \bar{\mu}_x + \frac{1}{2} \sigma_{L,x}^2 \right) (T_i - T_{i-1}) \right) \times \exp \left(\sigma_{L,x} (\tilde{W}_{x,T_i} - \tilde{W}_{x,T_{i-1}}) + \sum_{j=1+N_{T_{i-1}}}^{N_{T_i}} \ln(1 + Y_{1j}) \right), \tag{A.3}$$

where $(N_{T_i} - N_{T_{i-1}})$ can be generated by a Poisson distribution with intensity λ and jump size Y_{1j} . As the random sources of asset and liability processes, $dW_{x,t}$ and $d\tilde{W}_{x,t}$, are assumed to be correlated with that of the interest rate process, $dZ_{r,t}$, we set:

$$W_{x,T_i} - W_{x,T_{i-1}} = \rho_{x,r} (Z_{r,T_i} - Z_{r,T_{i-1}}) + \sqrt{1 - \rho_{x,r}^2} (Z_{x,T_i} - Z_{x,T_{i-1}}) \tag{A.4}$$

$$\tilde{W}_{x,T_i} - \tilde{W}_{x,T_{i-1}} = \tilde{\rho}_{x,r} (Z_{r,T_i} - Z_{r,T_{i-1}}) + \sqrt{1 - \tilde{\rho}_{x,r}^2} (\tilde{Z}_{x,T_i} - \tilde{Z}_{x,T_{i-1}}), \tag{A.5}$$

and construct $(Z_{x,T_i} - Z_{x,T_{i-1}})$ and $(\tilde{Z}_{x,T_i} - \tilde{Z}_{x,T_{i-1}})$ by normal random numbers.

Finally, the price of contingent capital (10) can be calculated via averaging over the simulated values of payoff (8) or (9).

Appendix B. Specification of asset dynamics

The term $Z_{x,t}$ is constructed, as a result of the projection, to be orthogonal to $Z_{r,t}$, and we now present the credit risk as in Duan et al. (1995):

$$dW_{x,t} = \rho_{x,r} dZ_{r,t} + \sqrt{1 - \rho_{x,r}^2} dZ_{x,t}, \tag{B.1}$$

where $\rho_{x,r} \equiv \frac{Cov(dW_{x,t}, dZ_{r,t})}{dt}$. Substituting (B.1) into (2) yields:

$$\frac{dA_{x,t}}{A_{x,t}} = r_t dt + \sigma_{A,x} \rho_{x,r} dZ_{r,t} + \sigma_{A,x} \sqrt{1 - \rho_{x,r}^2} dZ_{x,t}. \tag{B.2}$$

Using Eqs. (1) and (B.2) can be rearranged to yield:

$$\frac{dA_{x,t}}{A_{x,t}} = r_t dt + \frac{\sigma_{A,x}}{\sigma_r \sqrt{r_t}} \rho_{x,r} (dr_t - \kappa(\mu_r - r_t) dt) + \sigma_{A,x} \sqrt{1 - \rho_{x,r}^2} dZ_{x,t}. \tag{B.3}$$

Defining $\phi_{x,t} \equiv \frac{\sigma_{A,x}}{\sigma_r \sqrt{r_t}} \rho_{x,r}$ and $\psi_x \equiv \sigma_{A,x} \sqrt{1 - \rho_{x,r}^2}$, (B.3) can then be simplified as:

$$\frac{dA_{x,t}}{A_{x,t}} = (r_t - \phi_{x,t} \kappa(\mu_r - r_t)) dt + \phi_{x,t} dr_t + \psi_x dZ_{x,t}, \tag{B.4}$$

where $\phi_{x,t}$ demonstrates the instantaneous interest rate elasticity of x-company's assets that represents the stochastic modified duration.

The term $\tilde{Z}_{x,t}$ is similarly constructed to be orthogonal to $Z_{r,t}$. Projecting $d\tilde{W}_{x,t}$ onto $dZ_{r,t}$ yields:

$$d\tilde{W}_{x,t} = \tilde{\rho}_{x,r} dZ_{r,t} + \sqrt{1 - \tilde{\rho}_{x,r}^2} d\tilde{Z}_{x,t}, \tag{B.5}$$

where $\tilde{\rho}_{x,r} \equiv \frac{Cov(d\tilde{W}_{x,t}, dZ_{r,t})}{dt}$. In this study we assume that the random shocks of assets and liabilities are independent, and therefore $\tilde{Z}_{x,t}$ represents the pure liability risk that summarizes all continuous shocks that are not related to the interest rate or asset risk of x-company. Defining $\tilde{\phi}_{x,t} \equiv \frac{\sigma_{L,x}}{\sigma_r \sqrt{r_t}} \tilde{\rho}_{x,r}$ and $\tilde{\psi}_x \equiv \sigma_{L,x} \sqrt{1 - \tilde{\rho}_{x,r}^2}$ and substituting (B.5) into Eq. (3) yield:

$$\begin{aligned} \frac{dL_{x,t}}{L_{x,t}} &= (r_t - \lambda e^{\mu_x + \frac{1}{2}\sigma_x^2}) dt + \sigma_{L,x} \tilde{\rho}_{x,r} dZ_{r,t} + \sigma_{L,x} \sqrt{1 - \tilde{\rho}_{x,r}^2} d\tilde{Z}_{x,t} \\ &\quad + Y_{x,t} dN_t \\ &= (r_t - \lambda e^{\mu_x + \frac{1}{2}\sigma_x^2}) dt + \tilde{\phi}_{x,t} \sigma_r \sqrt{r_t} dZ_{r,t} + \tilde{\psi}_x d\tilde{Z}_{x,t} + Y_{x,t} dN_t \\ &= (r_t - \tilde{\phi}_{x,t} \kappa(\mu_r - r_t) - \lambda e^{\mu_x + \frac{1}{2}\sigma_x^2}) dt + \tilde{\phi}_{x,t} dr_t + \tilde{\psi}_x d\tilde{Z}_{x,t} \\ &\quad + Y_{x,t} dN_t. \end{aligned} \tag{B.6}$$

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