Designing Structure-Specified Mixed H_2/H_∞ Optimal Controllers Using an Intelligent Genetic Algorithm IGA

Shinn-Jang Ho, Shinn-Ying Ho, Ming-Hao Hung, Li-Sun Shu, and Hui-Ling Huang

***Abstract*—This brief proposes an efficient method for designing accurate structure-specified mixed H_2/H_∞ optimal controllers for systems with uncertainties and disturbance using an intelligent genetic algorithm (IGA). The newly-developed IGA with intelligent crossover based on orthogonal experimental design (OED) is efficient for solving intractable engineering problems with lots of design parameters. The IGA-based method without using prior domain knowledge can efficiently solve design problems of multi-input–multi-output (MIMO) optimal control systems, which is very suitable for practical engineering designs. High performance and validity of the proposed method are evaluated by two test problems, a MIMO distillation column model and a MIMO super maneuverable F18/HARV fighter aircraft system. It is shown empirically that the IGA-based method has good tracking performance, robust stability and disturbance attenuation for both controllers, compared with the existing methods.**

***Index Terms*—Intelligent genetic algorithm (IGA), mixed H_2/H_∞ optimal control, orthogonal experimental design (OED).**

I. Introduction

Mixed H_2/H_∞ optimal control design for systems with uncertainties and disturbance is an active area of research [1]–[10]. There are mainly two approaches to dealing with the mixed H_2/H_∞ optimal controller design problem; one is the structure-specified controller [1]–[7] and the other is the output-feedback controller [8]–[10]. The problem of designing a globally optimal full-order output-feedback controller for polytopic uncertain systems is known to be a nonconvex NP-hard optimization problem [10]. The techniques available in the literature for the output-feedback approach include branch-and-bound [8], convex upper bounds using semidefinite programming [9], bilinear matrix inequalities (BMIs) [10], etc. A new approach to the design of *locally optimal* output-feedback controllers via local BMI optimization is proposed in [10].

Since the order of the output-feedback controller is much higher than that of the plant, it is not easy to implement the controller for high-order systems in practical engineering applications [2]. To cope with this difficulty, the structure-specified approach solves the mixed H_2/H_∞ optimal control problem from suboptimal perspective. The investigated problem of structure-specified mixed H_2/H_∞ optimal control design is characterized by 1) nonlinear multimodal search space, 2) large-scale search space, 3) tight constraint, and 4) expensive objective function evaluation. This brief aims to develop a *practical* and efficient method for economically obtaining a potentially good approximation to a *globally optimal* solution to the investigated problem.

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TABLE I
History Chain of Researches on Structure-Specified Mixed H_2/H_∞ Optimal Control Problems

Ref.	Year	Problem	Input-Output type	Controller	No. of design parameters	Using domain knowledge	Algorithm
[1]	1995	mixed H_2/H_∞	SISO	PID	3	No	SGA
[2]	1998	H_∞	MIMO	PI	18	Yes	SGA
[3]	2001	mixed H_2/H_∞	SISO	PID	3	No	SGA
[4]	2001	H_∞	MIMO	PI	18	No	SGA
[5]	2002	mixed H_2/H_∞	MIMO	PID	12	No	Riccati
[6]	2003	mixed H_2/H_∞	SISO	PID	2-4	No	SGA
[7]	2004	mixed H_2/H_∞	MIMO	PID	18, 27	No	OSA
Ours	2004	mixed H_2/H_∞	MIMO	PID	12, 18, 27	No	IGA

Evolutionary computation is a robust search and optimization methodology, which is able to cope with ill-behaved problem domains, exhibiting attributes such as multimodality, discontinuity, time-variance, randomness, and noise [11]. A survey of evolutionary algorithms in control system engineering can be found in [12]. Recently, researchers have become increasingly interested in the use of genetic algorithm (GA) as a means to design various classes of control systems [1]–[4], [6].

The history chain of researches on structure-specified mixed H_2/H_∞ optimal control problems is shown in Table I. Chen and Cheng [2] used simple GA (SGA) to design multi-input–multi-output (MIMO) H_∞ optimal controllers for practical applications, but their procedure needs prior domain knowledge, i.e., the Routh–Hurwitz criterion for decreasing the domain size of each design parameter. Kitsios [4] used a GA-based method blended with multiobjective characteristics to improve the method of [2]. Tan *et al.* [5] investigated the problem of mixed H_2/H_∞ MIMO optimal control design and proportional-integral derivative (PID) tuning for multivariable processes by Riccati equations and H_∞-based method. Recently, Ho *et al.* [7] used an orthogonal simulated annealing algorithm (OSA) to design MIMO optimal controllers. The performance of the OSA-based method is superior to those of MIMO optimal controllers [2], [4].

A newly-developed intelligent genetic algorithm (IGA) [13] with intelligent crossover based on orthogonal experimental design (OED) [14], [15] can effectively solve intractable engineering problems with lots of design parameters, such as com-

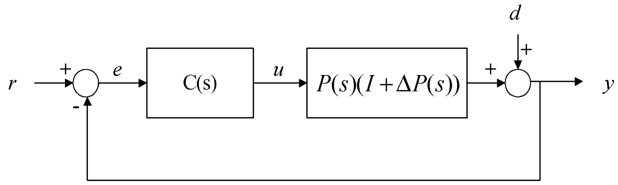


Fig. 1. Control system with plant perturbation and external disturbance.

binatorial optimization problem [16] and fuzzy classifier design [17]. The contribution of the brief is to propose an IGA-based method to obtain a near-optimal solution to the problem of designing structured-specified mixed H_2/H_∞ optimal controllers for systems with uncertainties and disturbance without domain knowledge and differentiability assumption. The proposed method is evaluated by two test problems, a MIMO distillation column model [5] and a MIMO super maneuverable F18/HARV fighter aircraft system [2], [4], [7]. It is shown empirically that the IGA-based method has good tracking performance, robust stability and disturbance attenuation, compared with the existing methods for MIMO optimal controllers [2], [4], [5], [7].

The remainder of this brief is organized as follows. Section II presents a problem description. Section III gives the IGA-based design method. Section IV gives two test problems to evaluate the proposed method. Finally, Section V concludes this brief.

II. Problem Description

Consider a MIMO control system with n_i inputs and n_o outputs as shown in Fig. 1, where $P(s)$ is the nominal plant, $\Delta P(s)$ is the plant perturbation, $C(s)$ is the controller, $r(t)$ is the reference input, $u(t)$ is the control input, $e(t)$ is the tracking error, $d(t)$ is the external disturbance, and $y(t)$ is the output of the system [2]. Without loss of generality, the plant perturbation $\Delta P(s)$ is assumed to be bounded by a known stable function matrix $W_1(s)$

$$\overline{\sigma}\left(\Delta P(jw)\right) \le \overline{\sigma}\left(W_1(jw)\right), \quad \forall w \in [0, \infty) \tag{1}$$

where $\overline{\sigma}(A)$ denotes the maximum singular value of a matrix A.

If a controller $C(s)$ is designed so that 1) the nominal closed-loop system ($\Delta P(s) = 0$ and $d(t) = 0$) is asymptotically stable, 2) the robust stability performance satisfies the following inequality:

$$J_a = \|W_1(s)T(s)\|_\infty < 1 \tag{2}$$

and 3) the disturbance attenuation performance satisfies the following inequality:

$$J_b = \|W_2(s)S(s)\|_\infty < 1 \tag{3}$$

then the closed-loop system is also asymptotically stable with $\Delta P(s)$ and $d(t)$. Where $W_2(s)$ is a stable weighting function matrix specified by designers. $S(s)$ and $T(s) = \mathrm{I} - S(s)$ are the sensitivity and complementary sensitivity functions of the system, respectively

$$\begin{aligned} S(s) &= (I + P(s)C(s))^{-1} \\ T(s) &= P(s)C(s)\left(I + P(s)C(s)\right)^{-1} \end{aligned} \tag{4}$$

and the H_∞-norm in (2) and (3) is defined as

$$\|A(s)\|_\infty \equiv \max_\omega \overline{\sigma}\left(A(j\omega)\right). \tag{5}$$

A balanced performance criterion to minimize both J_{a} and J_{b} simultaneously is to minimize the H_∞ norm value J_∞ [2], [4]: $J_\infty = (J_{\mathrm{a}}^2 + J_{\mathrm{b}}^2)^{1/2}$. For advancing the system performance, robust stability and disturbance attenuation are often not enough in the control system design. The minimization of tracking error J_2 (i.e., H_2 norm) should be taken into account

$$J_2 = \int_0^\infty e^{\mathrm{T}}(t)e(t)dt. \tag{6}$$

Where $e(t) = r(t) - y(t)$ is the error which can be obtained from the inverse Laplace transformation of $E(s)$ with $\Delta P(s) = 0$ and $d(t) = 0$

$$E(s) = (I + P(s)C(s))^{-1} R(s). \tag{7}$$

In the proposed method, the handling of constraints (2) and (3) is to recast the constraints as objectives to be minimized and, consequently, a weighted-sum approach is conveniently used. Therefore, the objective function of the investigated problem of designing mixed H_2/H_∞ optimal controllers is as follows:

$$\min_C J = J_2 + J_\infty. \tag{8}$$

A structure-specified controller of the following form [2]:

$$C(s) = \frac{N_c(s)}{D_c(s)} = \frac{\boldsymbol{B_m} s^m + \boldsymbol{B_{m-1}} s^{m-1} + \cdots + \boldsymbol{B_0}}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{9}$$

is assigned with some desired orders m and n to minimize J, where

$$\boldsymbol{B}_k = \begin{bmatrix} b_{k11} & \cdots & b_{k1n_i} \\ \vdots & \ddots & \vdots \\ b_{kn_o1} & \cdots & b_{kn_on_i} \end{bmatrix} \tag{10}$$

for $k = 0, 1, \ldots, m$. Most of the conventional controllers used in industrial control systems have fundamental structures such as PID and lead/lag configurations. Such controllers are special cases of the structure-specified controllers. A PI controller is a special case of the PID controller where $\boldsymbol{B}_2 = 0$. For the PID controller, we have $n = 1$, $m = 2$ and $a_0 = 0$, i.e.

$$C(s) = \frac{\boldsymbol{B}_2 s^2 + \boldsymbol{B}_1 s + \boldsymbol{B}_0}{s}. \tag{11}$$

III. IGA-Based Design Method

The proposed design method uses IGA with intelligent crossover based on OED. The used OED is briefly introduced in Section III-A. The intelligent crossover operation of IGA is described in Section III-B. Section III-C gives the IGA-based design method. The superiority of IGA and how to efficiently use IGA for solving various optimization problems can be referred to [13].

A. Used OED

An efficient way to study the effect of several factors simultaneously is to use OED with both orthogonal array (OA) and factor analysis [14], [15]. OED utilizes properties of fractional factorial experiments to efficiently determine the best combination of factor levels to use in design problems. OA is an array of numbers arranged in rows and columns where each row represents the levels of factors in each combination, and each column represents a specific factor that can be changed from each combination. The term "main effect" designates the effect on response variables that one can trace to a design parameter.

The two-level OA used in intelligent crossover is described in the following. Let there be N factors with two levels for each factor. The total number of level combinations is 2^N for a complete factorial experiment. To use an OA of N factors, we obtain an integer $M = 2^{\lceil \log_2(N+1) \rceil}$ where the bracket represents a ceiling operation, build a two-level OA $\mathrm{L}_M(2^{M-1})$ with M rows and $M-1$ columns, use the first N columns, and ignore the other $M-N-1$ columns. OA can reduce the number of combinations for factor analysis. The number of OA combinations required to analyze all individual factors is only M where $N+1 \leq M \leq 2N$. Algorithm of constructing the two-level OA can be found in [13].

For intelligent crossover, levels 1 and 2 of a factor represent selected genes from parents 1 and 2, respectively. After evaluation of the M combinations, the summarized data are analyzed using factor analysis. Factor analysis can evaluate the effects of individual factors on the objective (or fitness) function, rank the most effective factors, and determine the better level for each factor such that the function is optimized. Let y_t denote a function value of the combination t, where $t = 1, \ldots, M$. Define the main effect of factor j with level k as S_{jk} where $j = 1, \ldots, N$ and $k = 1, 2$

$$S_{jk} = \sum_{t=1}^{M} y_t \cdot F_t \tag{12}$$

where $F_t = 1$ if the level of factor j of combination t is k; otherwise, $F_t = 0$. Considering the case that the optimization function is to be minimized, the level 1 of factor j makes a better contribution to the function than level 2 of factor j does when $S_{j1} < S_{j2}$. If $S_{j1} > S_{j2}$, level 2 is better. If $S_{j1} = S_{j2}$, levels 1 and 2 have the same contribution. The main effect reveals the individual effect of a factor. The most effective factor j has the largest main effect difference $MED = |S_{j1} - S_{j2}|$. Note that the main effect holds only when no or weak interaction exists, and that makes the experiment meaningful. After the better one of two levels of each factor is determined, a reasoned combination consisting of N factors with better levels can be easily derived.

B. Intelligent Crossover

In the conventional crossover operations of GA, two parents generate two children with a random recombination of their chromosomes. The merit of intelligent crossover is that the systematic reasoning ability of OED is incorporated in the crossover operation to economically estimate the contribution of individual genes to a fitness function, and consequently intelligently pick up the better genes from two parents to form the chromosomes of children. The high performance of intelligent crossover arises from that intelligent crossover replaces the random recombination and generate-and-test search for children with the intelligent recombination using the systematic reasoning method.

A candidate solution consisting of p design parameters of an optimization problem is encoded into a chromosome using binary codes. If values of a specific parameter in two parent chromosomes are the same, this parameter is temporally unnecessary to participate in the intelligent crossover operation resulting in a smaller number of participated parameters. Let $L \leq p$ be the number of participated parameters in a parent chromosome. Using the same division scheme, divide two chromosomes of nonidentical parents into $N > 1$ pairs of nonoverlapping gene segments. The $N-1$ cut points are randomly specified from the $L-1$ candidate cut points which separate individual parameters. One gene segment of a chromosome is regarded as a factor of OED. To efficiently use all columns of OA excluding the study of intractable interactions among design parameters, the commonly used OA is $L_{N+1}(2^N)$ and the largest value of N is equal to $2^{\lfloor \log_2(L+1) \rfloor} - 1$ where the bracket represents a floor operation. One intelligent crossover operation takes $N+2$ (linear) fitness evaluations to efficiently explore the search space of 2^N (exponential) combinations.

Two parents P_1 and P_2 breed two children C_1 and C_2 using intelligent crossover at one time. How to use OED with L parameters to achieve intelligent crossover is described as the following steps.

- Step 1) Use the OA $\mathrm{L}_{N+1}(2^N)$ where $N = 2^{\lfloor \log_2(L+1) \rfloor} - 1$
- Step 2) Let levels 1 and 2 of factor j represent the jth parameter of a chromosome coming from parents P_1 and P_2, respectively.
- Step 3) Evaluate the fitness values y_t for experiment t where $t = 2, \ldots, N+1$. Note that y_1 is the fitness value of P_1.
- Step 4) Compute the main effect S_{jk} where $j = 1, 2, \ldots, N$ and $k = 1, 2$.
- Step 5) Determine the better level for each factor. Select level 1 for the jth factor if $S_{j1} > S_{j2}$. Otherwise, select level 2.
- Step 6) The chromosome of C_1 is formed from the intelligent combination of the better genes from the derived corresponding parents.
- Step 7) Rank the most effective factors from ranks 1 to N. The factor with a large main effect difference has a high rank.
- Step 8) The chromosome of C_2 is formed similarly as C_1 except that the factor with the lowest rank adopts the other level.
- Step 9) Verify that C_1 and C_2 are superior to the N combinations and parents according to the fitness performance. If it is not true, select the best two combinations from these $N+2$ combinations and parents as the final children C_1 and C_2 for the elitist strategy.

C. Design of Controllers Using IGA

For convenience and simplicity, from the controller with (10), we denote

$$\begin{aligned}\theta &= [a_0 \cdots a_{n-1} b_{011} \cdots b_{01n_i} b_{021} \cdots b_{02n_i} \cdots b_{mn_on_i}]^T \\ &= [\theta_1, \ldots, \theta_p]^T \end{aligned} \tag{13}$$

as the controller parameter vector, where $p = n + (m+1) \times n_i \times n_o$ is the number of total design parameters. The feasible solution θ_i is encoded using a binary string where $\theta_i \in [-20000, 20000]$, $i = 1, \ldots, p$ [2], [4], [7]. The proposed IGA-based design method is described as follows.

Step 1) For a given plant $P(s)$, specify the bound $W_1(s)$, weighting function matrix $W_2(s)$, and the controller structure $C(s)$.

Step 2) Initialization: Randomly generate an initial population of N_{pop} individuals.

Step 3) Evaluation: Compute fitness values of all individuals.

Step 4) Selection: A conventional truncation selection is used that the best $(1 - p_s) \cdot N_{\text{pop}}$ individuals are selected to form a new population, where p_s is a selection probability. Let I_{best} be the best individual in the population.

Step 5) Crossover: Randomly select $p_c \cdot N_{\text{pop}}$ parents including I_{best} for performing intelligent crossover operations, where p_{c} is a crossover probability.

Step 6) Mutation: Apply a conventional bit-inverse mutation operation with a mutation probability p_{m} to the population. To prevent the best fitness value from deteriorating, mutation is not applied to the best individual.

Step 7) Termination test: If a prespecified number $G_{\max}$ of generations are achieved, then stop the algorithm. Otherwise, go to Step 3).

IV. Experiments

For comparisons with existing methods, two test problems from [2] and [5] are used to evaluate the performance of the IGA-based method. The control parameters of IGA are specified as follows: $N_{\text{pop}} = 10$, $p_s = 0.2$, $p_c = 0.5$, and $p_m = 0.005$. Note that due to the merit of intelligent crossover, IGA uses a smaller value of N_{pop} than conventional GAs. Let the string length for each of p parameters be $p_i = 20$ and then the length of each chromosome is $20p$. Ten independent runs are performed for each test problem.

A. Test Problem 1

Consider a highly coupled distillation column model studied in [5]

$$P(s) = \begin{bmatrix} \frac{-33.98}{(98.02s+1)(0.42s+1)} & \frac{32.63}{(99.6s+1)(0.35s+1)} \\ \frac{-18.85}{(75.43s+1)(0.30s+1)} & \frac{34.84}{(110.5s+1)(0.03s+1)} \end{bmatrix}. \quad (14)$$

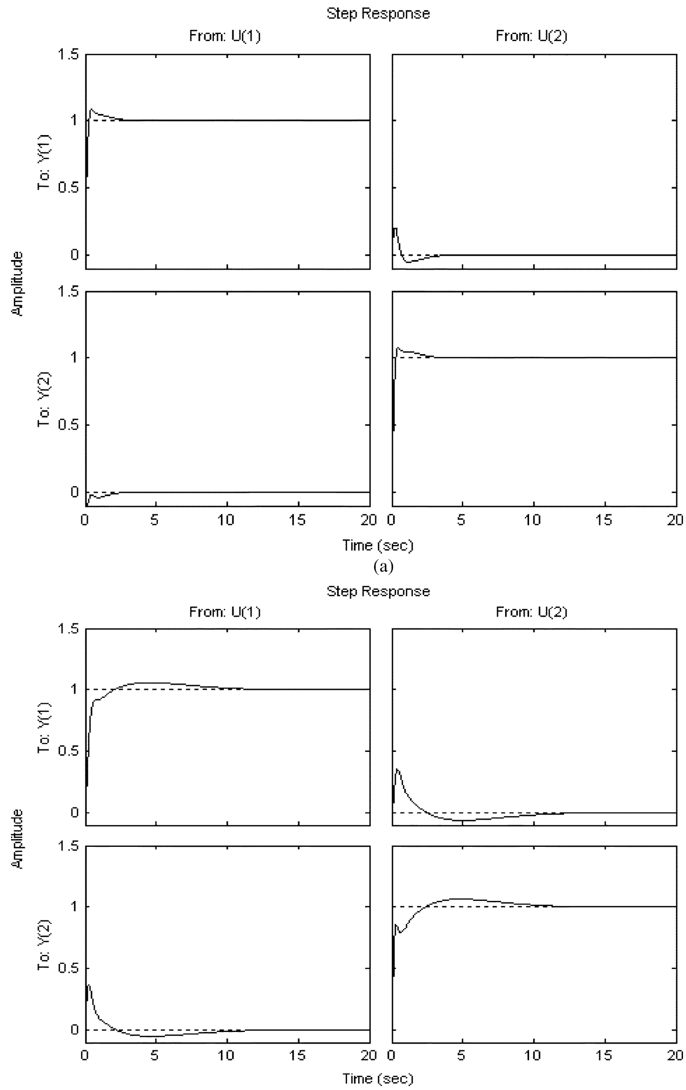
The bound $W_1(s)$ of the plant uncertainties $\Delta P(s)$ is

$$W_1(s) = \frac{100s+1}{s+1000} I_{2\times 2}. \quad (15)$$

To attenuate disturbance, a weighting function $W_2(s)$ consisting of a low-pass filter is

$$W_2(s) = \frac{s+1000}{1000s+1} I_{2\times 2}. \quad (16)$$

A typical controller $C(s)$ obtained from the IGA-based method with $G_{\max} = 1000$ as shown in (17) at the bottom of the next page.



(b)

Fig. 2. Output responses of the system for Problem 1 using various controllers. (a) IGA-based. (b) Riccati-based [5]. The left hand sides of (a) and (b) are the setpoint responses when setpoint changes at the first channel, and the other sides are those when setpoint changes at the second channel.

TABLE II
PERFORMANCE COMPARISONS OF PROBLEM 1 IN TERMS OF J_2 AND J_∞

PID Controller	J_2	J_∞	$J=J_2+J_\infty$	N_{eval}
Riccati-based [5]	42.6753	1.1588	43.8341	NA
IGA-based	16.7777	0.8520	17.6297	16211

This IGA-based PID controller is applied to the control system to illustrate the performance of the proposed method. The output responses of the system with the derived IGA-based and Riccati-based [5] controllers are shown in Fig. 2(a) and (b), respectively. The proposed PID controller has a smaller rising time and smaller coupling effects than the Riccati-based PID controller. The performance of the IGA-based method in terms of robust stability and disturbance attenuation (described by J_∞) and tracking error (described by J_2) is better than that of the Riccati-based method [5], as shown in Table II where N_{eval} denotes the number of used function evaluations.

B. Test Problem 2

For comparison with the methods proposed in [2], [4] and [7], the same MIMO optimal control design problem is tested. Consider the design problem of a longitudinal control system of the supermaneuverable F18/HARV fighter aircraft in horizontal flight at an altitude of 15 000 (ft) with Mach number 0.24, airspeed $V_T = 238.7$ (ft/s), attack angle $\alpha = 25$ (deg), and pitch angle $\beta = 25$ (deg). The trim value of the path angle is $\beta - a = 0$ (deg) and the trim pitch rate is $q = 0$ (deg/s). The longitudinal dynamics of the system can be described as

$$\left.\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned}\right\} \tag{18}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are given as (19), shown at the bottom of the page, and $\boldsymbol{x} = [V_T \ \alpha \ q \ \beta]^T$ and $\boldsymbol{u} = [u_{TV} u_{AS} u_{\mathrm{SS}} u_{LE} u_{\mathrm{TE}} u_T]^T$. Where u_{TV}, u_{AS}, u_{SS}, u_{LE}, u_{TE}, and u_T are the perturbations in symmetric thrust vectoring vane deflection, symmetric aileron deflection, symmetric stabilator deflection, symmetric leading edge flap deflection, symmetric trailing edge flap deflection, and throttle position, respectively. Note that the rank of the matrix $\boldsymbol{B}$ is only three. By employing the pseudo-control technique [18], we can transform the six control inputs (u_{TV}, u_{AS}, u_{SS}, u_{LE}, u_{TE}, and u_T) to three linearly independent variables. Therefore, the system can be rewritten as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_{\boldsymbol{v}}\boldsymbol{v} \tag{20}$$

where $\boldsymbol{B}_{\boldsymbol{v}}$ and $\boldsymbol{v}$ are given as (21), shown at the bottom of the page.

Suppose the reference inputs are $r(t) = [0,\ 1 - \mathrm{e}^{-3t},\ 1 - \mathrm{e}^{-6t}]^{\mathrm{T}}$ and the system is encountering with the external disturbance $d(t) = 0.01\mathrm{e}^{-0.2t}\cos(3162.3t)[1,\ 1,\ 1]^T$. The bound $W_1(s)$ of the plant perturbation $\Delta P(s)$ is

$$W_1(s) = \frac{0.0125s^2 + 1.2025s + 1.25}{s^2 + 20s + 100} I_{3\times 3}. \tag{22}$$

To attenuate disturbance, the stable weighting function $W_2(s)$ consisting of a low-pass filter is

$$W_2(s) = \frac{0.25s + 0.025}{s^2 + 0.4s + 10\,000\,000} I_{3\times 3}. \tag{23}$$

A typical PID controller $C(s)$ obtained from the IGA-based method with $G_{\max} = 50$ is (24), shown at the top of the next page. A typical PI controller $C(s)$ obtained from the IGA-based method with $G_{\max} = 150$ is (25), shown at the top of the next page.

Performance comparisons of various controllers in terms of J_2 and J_∞ are shown in Table III. The best OSA-based PID controller of Problem 2 has $J = 0.4393$ and $N_{\mathrm{eval}} = 2971$. The best IGA-based PID controller has $J = 0.1488$ and $N_{\mathrm{eval}} = 565$. Table IV shows the statistical results from the 10 runs of Problems 1 and 2. The simulation results illustrate that the IGA-based method can provide a very good solution to the problem of designing structure-specified mixed $\mathrm{H}_2/\mathrm{H}_\infty$ optimal controllers for systems with uncertainties and disturbance.

$$C(s) = \frac{\begin{bmatrix} -10.938 & -0.853 \\ -1.563 & -0.0002 \end{bmatrix} s^2 + \begin{bmatrix} -79.688 & -39.063 \\ -12.696 & 18.360 \end{bmatrix} s + \begin{bmatrix} -100.000 & -100.196 \\ -0.0006 & 1.775 \end{bmatrix}}{s}. \tag{17}$$

$$\boldsymbol{A} = \begin{bmatrix} -0.0750 & -24.0500 & 0 & -36.1600 \\ -0.0009 & -0.1959 & 0.9896 & 0 \\ -0.0002 & -0.1454 & -0.1677 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\boldsymbol{B} = \begin{bmatrix} -0.0230 & 0 & -0.0729 & 0.0393 & -0.0411 & 0.1600 \\ -0.0002 & -0.0001 & -0.0004 & 0 & -0.0003 & -0.0003 \\ -0.0067 & -0.0007 & -0.0120 & -0.0006 & 0.0007 & 0.0005 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

$$\boldsymbol{B}_{\boldsymbol{v}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{v} = \begin{bmatrix} -0.0230 & 0 & -0.0729 & 0.0393 & -0.0411 & 0.1600 \\ -0.0002 & -0.0001 & -0.0004 & 0 & -0.0003 & -0.0003 \\ -0.0067 & -0.0007 & -0.0120 & -0.0006 & 0.0007 & 0.0005 \end{bmatrix} \boldsymbol{u}. \tag{21}$$

$$C(s) = \frac{\begin{bmatrix} -10\,512.57 & 5906.282 & 588.742 \\ -10\,745.92 & -10\,313.56 & 473.919 \\ 401.402 & 692.921 & 15\,328.36 \end{bmatrix} s^2}{s} + \frac{\begin{bmatrix} -19\,779.82 & -976.048 & -341.244 \\ -5920.244 & -644.055 & 161.190 \\ 850.621 & 401.478 & 8434.246 \end{bmatrix} s + \begin{bmatrix} -15\,838.73 & 188.618 & 155.049 \\ -17\,755.43 & -17.452 & -596.600 \\ -709.592 & 1232.053 & -933.705 \end{bmatrix}}{s}. \quad (24)$$

$$C(s) = \frac{\begin{bmatrix} 268.116 & -0.515 & -7499.988 \\ -23.861 & -97.790 & -4257.454 \\ 0.820 & -7.916 & 9997.320 \end{bmatrix} s + \begin{bmatrix} 13\,626.588 & 12\,414.467 & -2913.001 \\ 9560.117 & -16\,754.643 & 488.759 \\ 17\,693.060 & -12\,531.769 & -4203.324 \end{bmatrix}}{s}. \quad (25)$$

TABLE III
PERFORMANCE COMPARISONS OF PROBLEM 2 IN TERMS OF J_2 AND J_∞

Controller	J_2	J_∞	$J=J_2+J_\infty$	N_{eval}
GA-based PI controller [2]	NA	0.8194	NA	18,000
GA-based PI controller [4]	0.1114	0.7682	0.8796	4,500
OSA-based PI controller [7]	0.0374	0.6299	0.6673	3,781
OSA-based PID controller [7]	0.0019	0.4374	0.4393	2,971
IGA-based PI controller	0.0158	0.6299	0.6457	3735
IGA-based PID controller	0.0092	0.1396	0.1488	565

TABLE IV
PERFORMANCE OF THE IGA-BASED CONTROLLER FROM 10 RUNS

Controller	J_2			J_∞			J		
	best	*avg.*	*std.*	*best*	*avg.*	*std.*	*best*	*avg.*	*std.*
Problem 1	16.7777	26.1121	5.5672	0.6957	0.8147	0.0845	17.6297	26.9268	5.5502
Problem 2 (PI)	0.0125	0.0261	0.0113	0.6299	0.6688	0.0252	0.6457	0.6949	0.0296
Problem 2 (PID)	0.0023	0.0263	0.0203	0.1396	0.2974	0.1574	0.1488	0.3237	0.1486

V. CONCLUSION

This brief proposes a method for obtaining a near-optimal solution to the problem of designing structure-specified mixed H_2/H_∞ optimal controllers for systems with uncertainties and disturbance using an IGA. The high performance and validity of the proposed method are demonstrated by two test problems, a MIMO distillation column model and a MIMO super maneuverable F18/HARV fighter aircraft system with PI and PID controllers. It is shown empirically that the performance of the IGA-based method without using specific problem-dependent strategies and knowledge is superior to those of some existing methods in terms of tracking error, robust stability, and disturbance attenuation. The IGA-based method can be most widely used for designing high-performance optimal controllers.

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